

A Math Trail at Dallas City Hall
walkSTEM@Dallas City Hall Plaza
Solutions to questions

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Dallas City Hall is actually a skyscraper placed on its side, anchored at the southern end of the CBD. It is bordered by Akard, Evray, Young and Canton Streets and adjoins the Convention Center site. The most dynamic aspect of this 770,000 sq. ft., cast-in-place concrete structure is its trapezoidal shape, dramatically defined by the cantilevered front face of the building. This overscaled trapezoid, situated at the rear half of a rectangular, ten acre site stretches 560 ft. in length and rises to a height of 122 ft. from ground level. The sloping front of the building faces north to the vertical skyscrapers of the CBD. As the building's face angles 34 degrees out and away from perpendicular, it increases the depth of the roof area to 194 ft., versus the 126 ft. depth of the base area at ground level. This design increases the depth of each of the above grade floors by 9 1/2 ft. in arithmetic progression, enabling an agency and its support staff to be located on a common floor (or a part of one), which is in ratio to their collective size, and which accommodates their total space requirements. The design, thereby structurally imitates and facilitates the operations of the city's government, a lesson in form follows function instilled by Gropius. There are eight floors above grade, the first seven of which provide public areas and office space; the eighth houses the building's mechanical equipment. Two floors of additional office space extend below grade and abutt the two-level, 1,325 car parking facility beneath City Hall's park plaza. The east and west facades (the mirrored-L-shaped ends) and the rear south face, cantilever outward, but these faces of the building are stepped-out, overhanging each preceding floor, instead of forming a smooth inclined plane, as is represented by the front face.

Five Buildings in the Dallas Central Building District by I. M. Pei and Partner Henry N. Cobb: A Stamp on the City's Direction

Thesis presented to the Graduate Council of the North Texas State University in Partial Fulfillment of the Requirements For the Degree of Master of Arts by J. Barney Malesky, M. A., Denton, Texas, December, 1986

https://digital.library.unt.edu/ark:/67531/metadc500988/m2/1/high_res_d/1002775355-Malesky.pdf

Question 1

- (a) Why did Pei use this sloping design for the city hall building? How is the form related to the function of the building?
- (b) Use an app that measures angles such as Angle Finder, Angle Meter, Bubble Level or Clinometer to verify that the building slopes at an angle of 34° .
- (c) If Pei decided to make each above grade floor 4.75 feet wider than the one below, would the building slope at an angle of 17° ? How do you know?
- (d) Use the information contained in the excerpt from J. Barney Malesky's thesis and the scale diagram given below (Figure 1) to calculate the height of a floor.

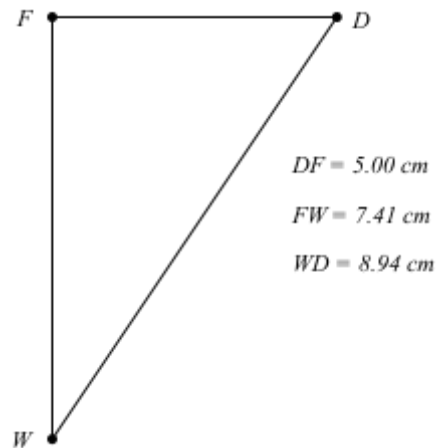


Figure 1

- (e) Use your answer to part (d) to determine the height of City Hall. Compare your result with the figure stated in the excerpt from J. Barney Malesky's thesis. Are both heights the same? If not, why not?
- (f) Calculate the width and square footage for each of the seven floors which provide public areas and office space plus the mechanical floor on the 8th floor (Table 1). Determine the square footage of the two floors of additional office space that extend below grade.

- (g) This question requires some basic knowledge of trigonometry. In Figure 2, how are the numbers 2 and 3 that appear on the right side about halfway down, related to the values of $\tan(34^\circ)$ and $\tan(56^\circ)$?

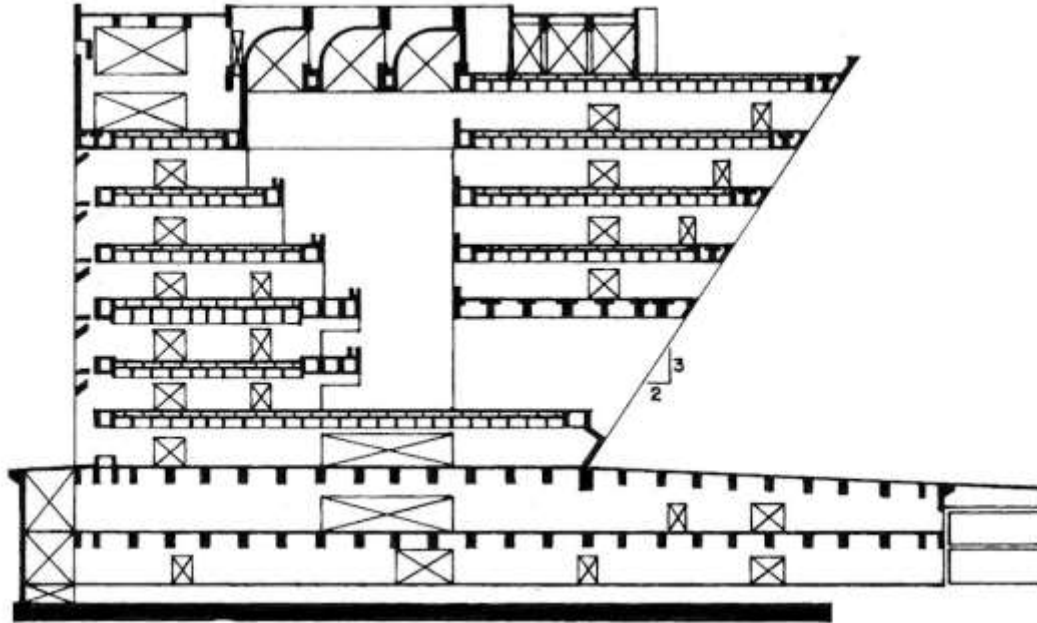


Figure 2

http://www.architectureweek.com/cgi-bin/awimage?dir=2009/0729&article=building_2-2.html&image=14298_image_6.jpg

Question 2

Walk to the "The Dallas Piece", a sculpture by Henry Moore (Photograph 1).

- (a) In 1976, the City of Dallas commissioned Henry Moore to create a sculpture for the plaza. How does this sculpture relate to the City Hall building? Do you feel that it fits in with the building and the plaza? Why or why not?

After you have thought about the role of the sculpture, visit the website given below to learn more about Moore's reasons for his design and what he was trying to do in terms of the plaza and the building.

<http://www.publicartarchive.org/work/dallas-piece>

- (b) The length of the sculpture does not appear in the text given below. Suggest a reason for why it was not included. Estimate the length.

A 16-foot (4.9 m)-high by 24-foot (7.3 m)-wide, three-piece sculpture titled "The Dallas Piece" was designed by Henry Moore for the plaza and resembles vertebrae.

https://en.wikipedia.org/wiki/Dallas_City_Hall



Photograph 1 by Ron Lancaster: "*Dallas Piece*" by Henry Moore

Question 3

Walk to the nearby fountain and read the text given below.

The Park Plaza is two blocks long and one block wide and is bounded by Young, Ervay, Marilla and Akard streets. The Plaza includes a 180-foot (55 m)-diameter reflecting pool, a variable-height fountain, park benches and three distinctive 84-foot (26 m)-high flagpoles. The Plaza is landscaped with trees native to Texas: live oaks and red oaks. The reflective pool contains large floating sculptures designed by artist Marta Pan.

https://en.wikipedia.org/wiki/Dallas_City_Hall

- (a) When you look at the fountain and City Hall, does the circumference of the fountain appear to be more, less or the same as the length of the building?

- (b) Starting at a point P on the edge of the fountain (Figure 3), walk around the fountain while someone else walks from one end of City Hall to the other (point A to point B). How can you use the time it takes both of you to do your walk to compare the circumference of the fountain with the length of the City Hall building? This is assuming that both of you walk at the same rate. How will you actually do that?

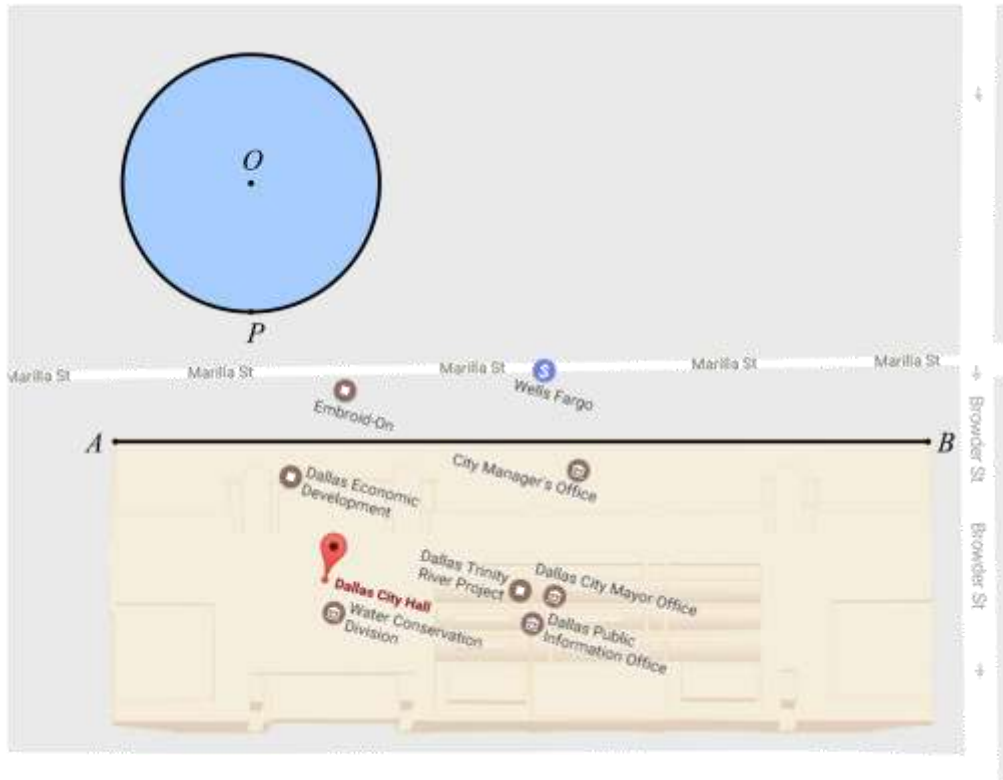


Figure 3

Question 4

Return to the point where you started your walk and have a friend stand beside one of the tall flag poles. Walk around the fountain in a counter-clockwise direction several times at a constant rate while your friend stands still. Let Y represent your location, as you walk, and F the position of your friend (Figure 4). The point P is your starting point.

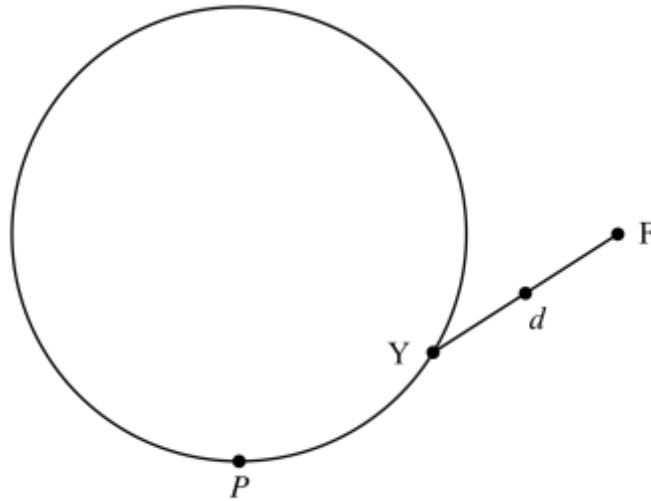


Figure 4

- (a) Let d represent the distance (in feet) between the two of you (the length of line segment YF) and let t represent the time (in seconds). Graph the distance d versus the time t . Since you have no actual data to work with, a precise graph is not required.
- (b) How would the graph in part (a) change if you walked at a faster rate?
- (c) How would the graph in part (a) change if, before you began to walk, your friend moved to a point further away from the fountain to be in the shade?

Question 5

- (a) Describe the movement of one of the red sculptures designed by the artist Marta Pan (Photograph 2).
- (b) Suppose that the sculpture is anchored to the bottom of the fountain and that its movement is controlled by a motor that rotates it through 360° every 5 seconds. How many complete rotations will the sculpture go through in an hour? a day? a year?
- (c) Suppose that Marta Pan made this sculpture by starting with a sphere. Estimate the percentage of the sphere that was removed in making the sculpture. Suggest a method for calculating this percentage.



Photograph 2 by Ron Lancaster: Floating Sculpture by Marta Pan

Question 6

- (a) Walk a short distance to the three massive flag poles. Each one of them is an example of a tapered flagpole that forms a cone. What other types of flagpoles are there? Why do you think tapered flagpoles were selected for the City Hall plaza?

For the following questions you will need to use the formulas for the volume of a cone and a cylinder provided below.

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h \quad \text{Volume of a cylinder} = \pi r^2 h$$

- (b) Estimate the circumference at the base of one of the flagpoles and calculate the volume of the flagpole (cone).
- (c) Suppose that these tapered flagpoles were cylinders with the same volume. Would they be shorter or taller than they are now? How much shorter or taller would they be?

Question 7

Read the information given below and then answer the following questions.

Finding the right ratio of flag to pole

I see a lot of poles flying the wrong sized flag. You've probably noticed, too. Some tall poles have very small flags, and vice versa. So how do you determine the correct size flag to put on your pole?

Here's my rule of thumb. You fly a flag that looks right; that has the right proportions. Your eye can tell just by looking, if the flag is large enough, but, you still ask, where do you start?

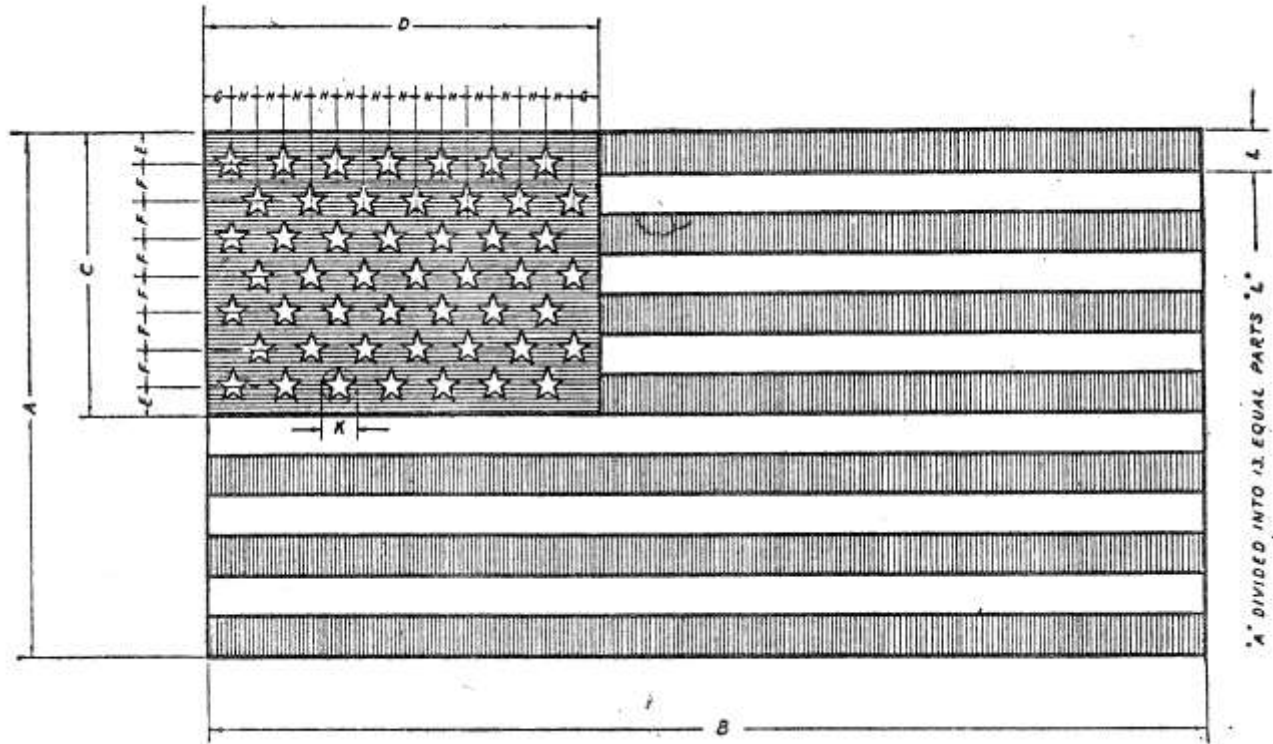
At 20%. Yep! If the hoist of your flag (or the height of the flag) is 20% of the height of the pole, you are in good shape. And since the numbers aren't always exact, some judgment is required.

<https://www.thedailyflag.com/2008/07/09/finding-the-right-ratio-of-flag-to-pole/>

- (a) Does the size of the US flag on the flagpole in the City Hall plaza look "right" to you?
- (b) Suppose the height of the US flag is 20% of the height of the pole. Calculate the height of the flag.

Question 8

Use the official specifications for the US flag given below, to calculate the height and width for a US flag for which the diameter of the star is equal to the diameter of the fountain. If the 20% rule of thumb is used, how tall would the flagpole be? How would it compare to the tallest building in Dallas?



https://commons.wikimedia.org/wiki/File:1959_US_Flag_specification.jpg

Hoist (height) of the flag: $A = 1.0$

Fly (width) of the flag: $B = 1.9$

Diameter of star: $K = 0.0616 (L \times 4/5, \text{four-fifths of the stripe width})$

Width of stripe: $L = 0.0769 (A/13, \text{One thirteenth of the flag height})$

These specifications are contained in an executive order which, strictly speaking, governs only flags made for or by the U.S. federal government. In practice, most U.S. national flags available for sale to the public have a different width-to-height ratio. Flags that are made to the prescribed 1.9 ratio are often referred to as "G-spec" (for "government specification") flags.

https://en.wikipedia.org/wiki/Flag_of_the_United_States

Question 9

The flag of the state of Texas is defined by law as follows:

The state flag is a rectangle that: (1) has a width to length ratio of two to three; and (2) contains: (A) one blue vertical stripe that has a width equal to one-third the length of the flag; (B) two equal horizontal stripes, the upper stripe white, the lower stripe red, each having a length equal to two-thirds the length of the flag; and (C) one white, regular five-pointed star: (i) located in the center of the blue stripe; (ii) oriented so that one point faces upward; and (iii) sized so that the diameter of a circle passing through the five points of the star is equal to three-fourths the width of the blue stripe.

https://en.wikipedia.org/wiki/Flag_of_Texas

- (a) Does the flag of Texas on the flagpole in the City Hall plaza appear to adhere to the regulations given above?
- (b) Suppose that the length and width of an official flag of Texas are 24 inches and 16 inches (Figure 5). Calculate the area of the white circle and the area of the white rectangle DEFH. Show that the area of the circle is slightly less than 25% of the area of the rectangle DEFH. Determine the diameter of the circle for which its area is exactly equal to 25% of the area of rectangle DEFH.

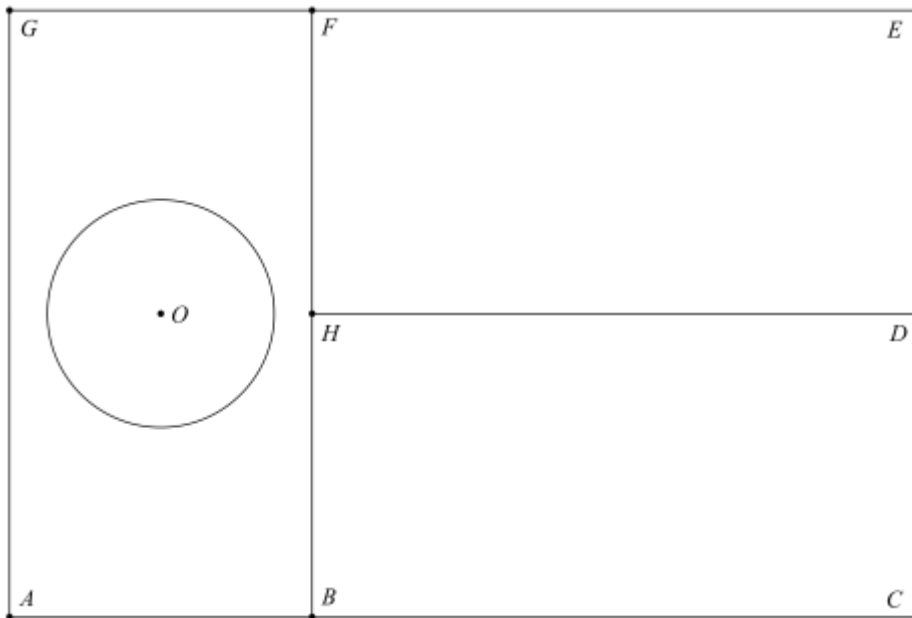


Figure 5

Question 10

Figure 6 shows the flag from Figure 5 on a set of coordinate axes with point A positioned at (0, 0).

(a) Determine the coordinates of point O, the center of the circle. What is the equation of the circle?

(b) Determine the coordinates of the points P, Q, R, S and T that determine the star.

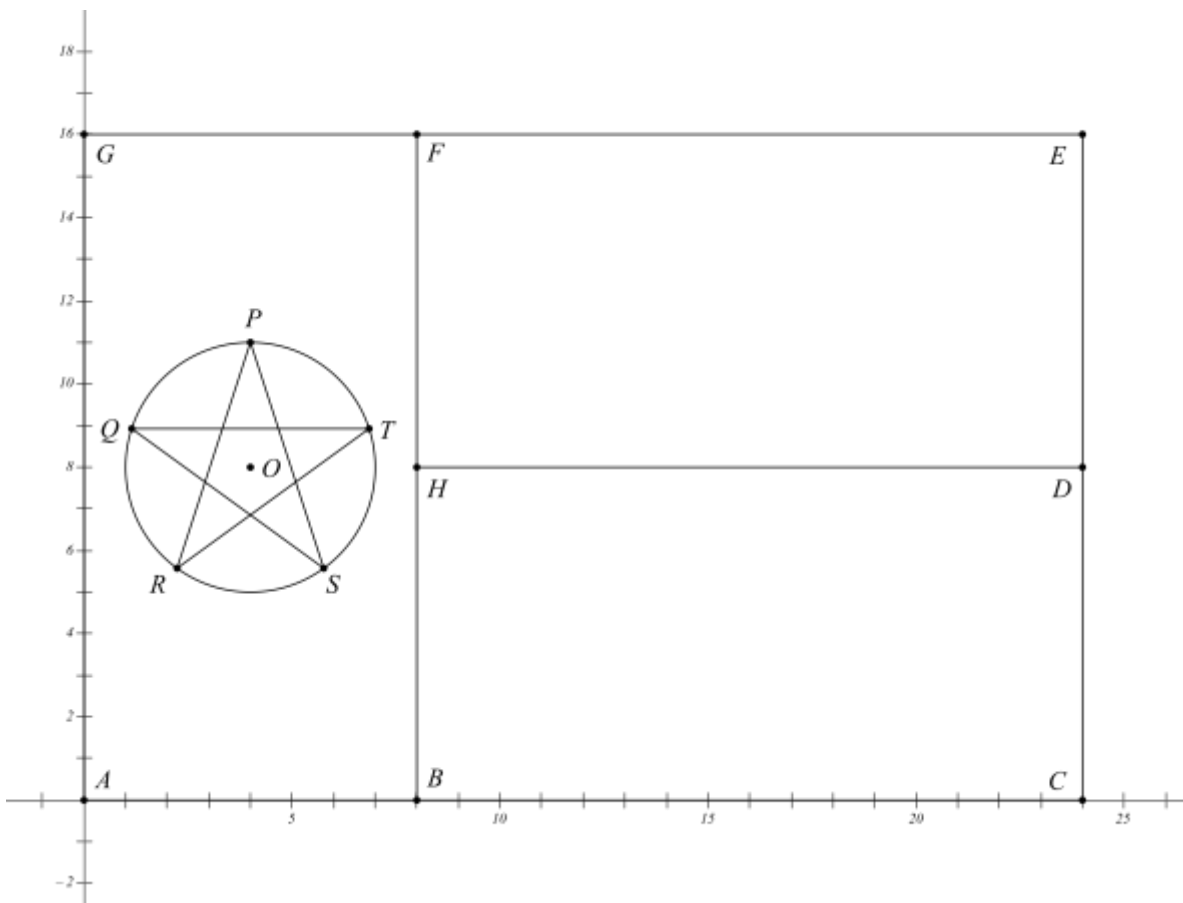


Figure 6

Question 11

Walk down the nearby steps towards Young Street, turn right and locate the City Seal for the City of Dallas shown in Photograph 3.

- (a) This photograph was taken on Saturday, April 8, 2017. What time of the day was it taken? How do you know?

- (b) What mathematical questions do you have about the City Seal?



Photograph 3 by Ron Lancaster: City Seal for the City of Dallas.

The end!

Here are two puzzles for you to work on after the Math Trail.

Question 12

In the addition puzzle given below, each letter represents a digit from 0 to 9. Any given letter stands for just one digit (in other words the value of C cannot be say 1 and 4 at the same time) and two different letters cannot stand for the same digit (so I and T can't both be say 5). Also the first letter of any word cannot be 0. There are 24 solutions to this puzzle, one for each hour of the day. Take a minute and find one of them and then find a second in a second by switching the values for C and M.

$$\begin{array}{rcccc}
 C & I & T & Y \\
 H & A & L & L \\
 M & A & T & H \\
 \hline
 P & A & T & H
 \end{array}$$

Question 13

The solution to the addition puzzle given below has no connection to the solution to the puzzle in Question 12. It is not a continuation of Question 12 in the sense that the value of a letter carries over to this puzzle. Coincidentally, there are also 24 solutions to this puzzle. Find at least one of them.

$$\begin{array}{rcccc}
 S & T & E & M \\
 T & A & L & K \\
 & & A & T \\
 & T & H & E \\
 M & A & T & H \\
 \hline
 W & A & L & K
 \end{array}$$

Bibliography

Websites

MfA Math Trail at the Rubin Museum of Art
<https://www.youtube.com/watch?v=6KZ8KOWHSWc>

Math for America Math Trails
<https://vimeo.com/6085446>

The Geometry Playground: Walk a Math Trail
<http://www.exploratorium.edu/geometryplayground/mathtrails.php>
<http://tinyurl.com/walkamathtrail>

The Canadian Math Trail
<http://www.brocku.ca/cmt/English/index.htm>

Vancouver 2010 Olympic Math Trail
<http://www.brocku.ca/cmt/upload/1070004370.8764/index.html>

Math Trails
<http://www.cmste.uregina.ca/MathTrails/>

Welcome to the Welland Canal Math Trail
<http://spartan.ac.brocku.ca/~emuller/mathtrail/wcmt/wcmtstart.html>

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Tales about my Trails by Ron Lancaster

I created my first Math Trail on Toronto's Centre Island in the summer of 1985 for teachers who were enrolled in an Additional Qualifications (AQ) course I taught at the Faculty of Education of the University of Toronto. The suggestion to go to Centre Island came from Al Fleming, a well known mathematics teacher, who had developed a tradition of going to the island for the last class of his AQ courses. In the early 1990s I began to create Trails for teachers who attended the Phillips Exeter Academy Mathematics, Science and Technology Conference in Exeter, New Hampshire. I developed these Trails with Vince Delisi, a mathematics teacher from Toronto. Vince and I created Trails on the Exeter campus, in the downtown area and in stores including Whirlygigs Toys and Exeter Candy.

In 1996 I began to develop Trails for my students at St. Mildred's-Lightbourn, an all-girls school in Oakville. The first Trail that I created was set in the Financial District of Toronto and the day was a major success. The students loved the active nature of the project, they enjoyed being able to work collaboratively and they were stunned at how much mathematics there is in the world around us. The following year my grades 7 and 8 students participated in three Trails. The first Trail took place in Toronto and I used it as a way of getting to know my students and as a way for my students to be exposed to the big ideas of the course. The second Trail was on the campus of the school and I used it to help students practice what they had learned to date. The third Trail was also in Toronto and this one was used to help students review everything we had done during the year. The third Trail also contained questions from the curriculum for the next year in an effort to give students a sense for what was to come.

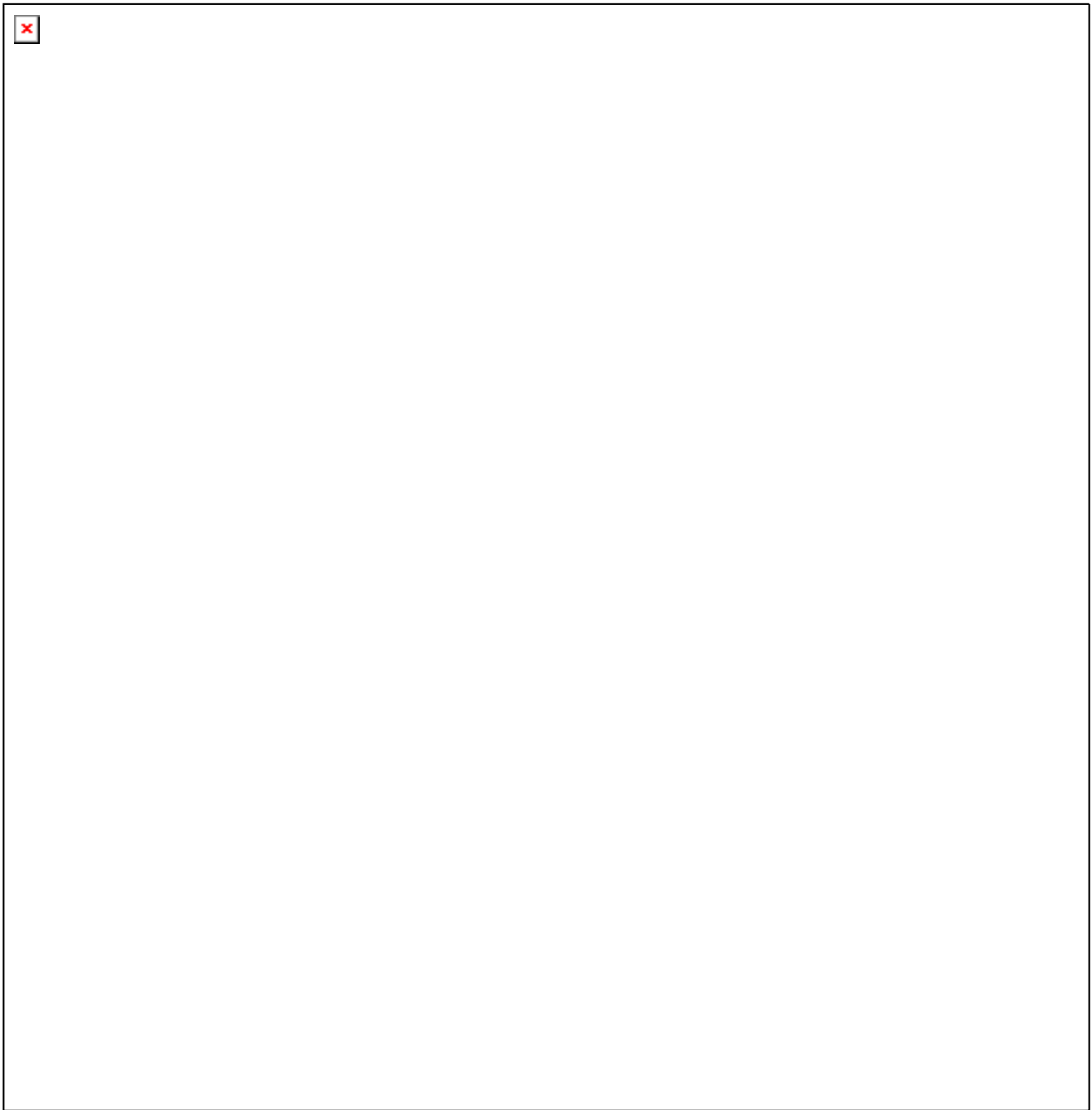
I have created other Math Trails in numerous cities in Canada and the US; Abu Dhabi; Accra; Bangkok; Delhi; Hong Kong; Nanjing, Suzhou and in Singapore where over 7000 students and hundreds of teachers have enjoyed these walks. I have also developed Math Trails for the Avenues School, the Museum of Mathematics and Math for America in New York City at the Museum of Modern Art, the Museum of Natural History, Ellis Island, the Bronx Zoo, Madison Square Park, the campuses of New York University & Columbia University and on the High Line.

There have been other people around the world who have developed their own Trails. Kay Toliver is one of the best-known Trail blazers and information about her work can be found at <http://thefutureschannel.com/kay-toliver/>. Many years ago I worked with Kay in Los Angeles and New Orleans and she was a pleasure to be with. Geoff Kavanagh, an Instructor at the University of Toronto, was a real pioneer in the development of Math Trails and he was out with his students in downtown Toronto in the 1980s. Eric Muller, a Mathematics Professor at Brock University in Canada was another early Trail blazer. In the 1980s, he developed Math Trails in Niagara Falls and at the Welland Canal.

Other trail developers include close friends of mine such as Brigitte Bentele, who has developed Trails in Manhattan; Larry Ottman, Lisa Ledwith and Marcia Wexler who have developed Trails in Philadelphia; Diane Devine who has developed Trails in Boston and Carly Ziniuk who has developed Trails in Toronto and in Ottawa at the National Gallery of Canada.

And of course there is Glen Whitney, the founder of the Museum of Mathematics, who has developed math walks around the US and is a passionate path developer.

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Ron Lancaster at Dallas City Hall: Angle shot by Koshi Dhingra, April 2017